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Taming polar active matter with moving substrates: directed transport and counterpropagating macrobands

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Supplementary material for this article is available [online](#)

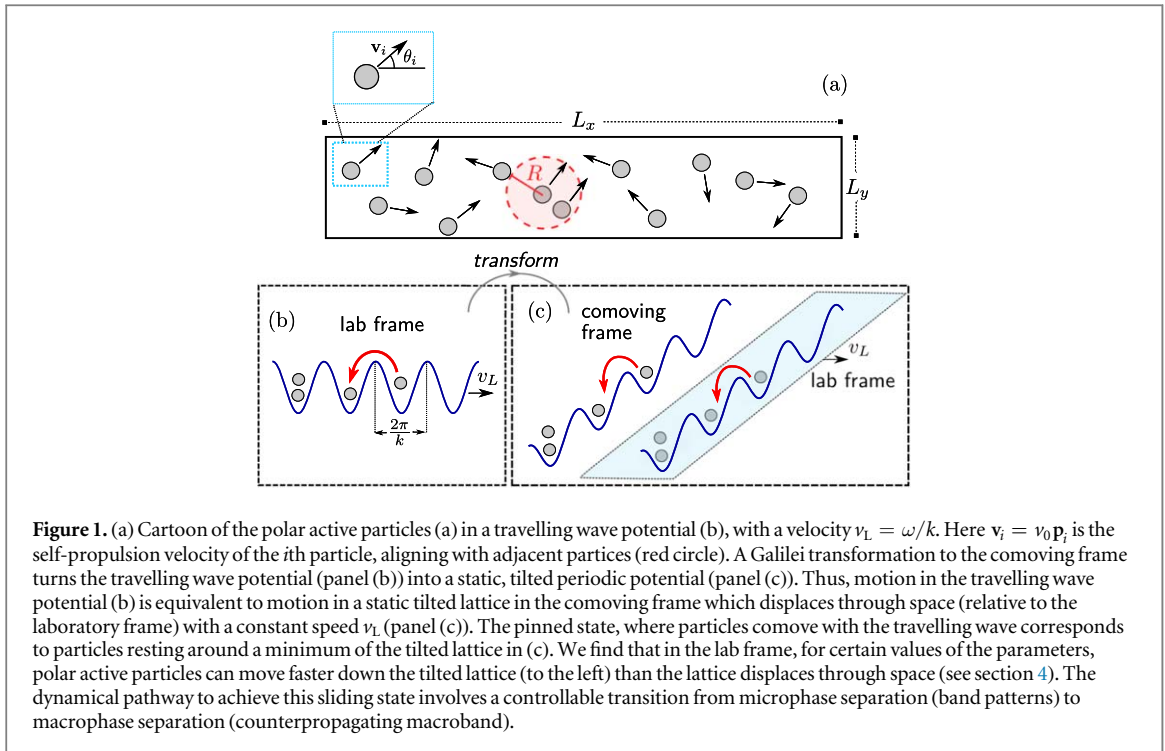
Abstract

Following the goal of using active particles as targeted cargo carriers aimed, for example, to deliver drugs towards cancer cells, the quest for the control of individual active particles with external fields is among the most explored topics in active matter. Here, we provide a scheme allowing to control collective behaviour in active matter, focusing on the fluctuating band patterns naturally occurring e.g. in the Vicsek model. We show that exposing these patterns to a travelling wave potential tames them, yet in a remarkably nontrivial way: the bands, which initially pin to the potential and comove with it, upon subsequent collisions, self-organize into a macroband, featuring a predictable transport against the direction of motion of the travelling potential. Our results provide a route to simultaneously control transport and structure, i.e. micro- versus macrophase separation, in polar active matter.

1. Introduction

Active matter contains self-propelled particles like bacteria, algae, or synthetic autophoretic Janus colloids whose properties can be designed on demand [1–3]. As one of their main characteristics, these systems are intrinsically out of equilibrium allowing them to self-organize into new ordered and even functional structures. In synthetic active systems, such structures include dynamic clusters which dynamically form and break-up in low density Janus colloids [4–8] as well as laser driven colloids which spontaneously start to move ballistically (self-propel) when binding together [9–11]. Likewise, biological microswimmers form patterns such as vortices in bacterial turbulence [12–15], or swirls and microflock patterns in chiral active matter like curved polymers or sperm [16–19].

Much of what we know about active systems and the patterns they form roots in explorations of minimal models which to some extent represent broader classes of active systems showing the same symmetries. The pioneering example of such a minimal model is the Vicsek model describing polar self-propelled particles such as actin-fibres mixed with motor proteins [20, 21], certain microorganisms [22], self-propelled rods [23, 24] or ‘birds’ [22, 25] which only see their neighbours and have a tendency to align with them, in competition with noise. While forbidden in equilibrium [26] the Vicsek model shows true long-range order in two dimensions [25], meaning that activity makes orientational correlations robust against noise over arbitrarily long distances. The phase transition from the disordered phase which occurs for strong noise to the long-range ordered Toner–Tu phase is now known to be discontinuous [27] and features a remarkably large coexistence region [28, 29] where high-density bands of comoving polarized particles spontaneously emerge and traverse through a background of a low-density disordered gas-like phase. These bands behave highly randomly; they merge when colliding with each other but also split up frequently, rendering an irregular pattern of sharply localized and strongly polarized moving bands. The latter choose their direction of motion spontaneously depending on initial state and fluctuating molecular environment (noise realization); thus, when averaging over many realizations, there is no net motion. This randomness is unfortunate in view of potential key applications of



active matter, e.g. for targeted drug delivery, crucially requiring schemes to control active particles. Here, while single particle guidance with external fields is among the most explored problems in active matter [5, 30–37] and there is a moderate knowledge on interacting particles in external fields (complex environments) [38–43] and their control [44–48], surprisingly little is known about the controllability of polar active particles and the band patterns they naturally form.

In the present work we ask for a scheme to tame band patterns, i.e. if we can force the bands in the Vicsek model to settle down into a pattern featuring a predictable and externally controllable direction of motion. To achieve this, we apply a ‘travelling wave potential’ (also called travelling potential ratchet [49]) to the Vicsek model. We find that such an external field does in fact allow to control the late time direction of motion of particle ensembles in polar active matter, yet, in a remarkably nontrivial way. In our simulations, for appropriate parameter regimes, we see the formation of bands that at early times pin to the minima of the travelling potential and comove with it. When time proceeds, one of the bands suddenly unpins and starts counterpropagating in the travelling potential. Upon subsequent collisions the band swells towards a macroband containing most particles in the system. This macroband emerges representatively in a large parameter window and shows a predictable motion against the travelling direction of the potential. Our results show that a moving (or tilted, see figure 1) substrate tames the collective behaviour of polar active particles and can be used to control the transition from microphase separation (band patterns) to a macrophase separated state which does show predictable transport. In the following, we specify these results and analyze the mechanism underlying the emergence of a counterpropagating macroband.

2. Model

We consider $N = 5000$ active overdamped particles in a quasi-1D-potential $V(x, y, t)$, which is uniform in y -direction and represents a travelling wave in x -direction; i.e. it is periodically modulated and moves with constant speed $v_L = \omega/k$ in x -direction, where ω, k are the frequency and wave vector of the travelling potential (figure 1(b)). Such a potential has previously been considered for active point particles [42] and disks [50] and can be realized e.g. by a micropatterned ferrite garnet film substrate [51], by optical lattices traversing at speeds of a few μ/s , or effectively (see figure 1(c)), simply by a tilted washboard potential [52–54]. Note here that in the comoving frame (moving with a constant velocity v_L) the dynamics translates into motion of a particle in a static tilted washboard potential (see figures 1(b), (c) and section 4).

Besides experiencing the external potential, the active particles also self-propel, a fact effectively described by a self-propulsion force $\gamma v_0 \mathbf{p}_i$ where $\mathbf{p}_i = \cos \theta_i \mathbf{e}_x + \sin \theta_i \mathbf{e}_y$; $i = 1, \dots, N$ are the self-propulsion directions of the particles and γ is the Stokes drag coefficient. In bulk, the particles would move with a constant speed v_0 . As in the Vicsek model, we assume that the particles align with each other. We define the dynamics of the particles by

the following equations of motion³:

$$\dot{\mathbf{r}}_i = v_0 \mathbf{p}_i + \mathbf{F}_i/\gamma \quad (1a)$$

$$\dot{\theta}_i = \frac{g}{\pi R^2} \sum_{j \in \delta_i}^N \sin(\theta_j - \theta_i) + \sqrt{2D_r} \eta_i(t). \quad (1b)$$

Here, g controls the strength of alignment of a particle with its neighbours within a range R and the sum is performed over all these neighbours (see figure 1(a)). Alignment competes with rotational Brownian diffusion, occurring with a rate D_r ; η_i represents Gaussian white noise of zero mean and unit variance. The force due to the substrate reads

$$\mathbf{F}_i := -\nabla U = \gamma u_0 \cos(2\pi(kx_i - \omega t)) \mathbf{e}_x, \quad (2)$$

where u_0 is the strength of the external force. Here, in all of our results we express lengths and times in units of μm and s respectively, i.e. we introduce parameters $D_r' = D_r \cdot s$, $g' = g \cdot s/\mu\text{m}^2$ etc and omit primes for simplicity, allowing thus for a straightforward comparison with potential experiments. Nevertheless, the actual dimensionless parameters of our setup can also be easily extracted on demand⁴.

We now study the dynamics of the described model using Brownian dynamics simulations and an elongated simulation box of size $L_x \times L_y = 500 \times 5$, fixing the density to $\rho = 2$, as well as random but uniformly distributed initial particle positions and orientations. Using more quadratic boxes or different system sizes leads to qualitatively similar phenomena (see supplementary video 1 available online at stacks.iop.org/NJP/21/013023/mmedia for the case of a more quadratic box). Also the specific value of the overall particle density ρ seems to be rather unimportant as long as the system stays in the large parameter regime where band patterns emerge.

3. Counterpropagating macroband

In the absence of a lattice, our simulations reveal the usual phenomenology of the Vicsek model [27–29]: for a given alignment strength ($g = 0.07$) and comparatively strong noise $D_r > D_r^c \approx 0.15$ (or high temperatures), we find a disordered uniform phase (figures 2(a) and (b)), whereas noise values $D_r < D_r^c$ lead to a polarized phase (figures 2(c) and (d)). In the latter phase, particles self-organize into polarized bands of high density which move with a speed $\sim v_0$ and coexist with gas-like unpolarized regions in between the bands. The bands occur at seemingly irregular distances to each other. As time proceeds, they occasionally split up (for $D_r \neq 0$) and typically merge when they collide with each other; overall, the number and size of the bands changes dynamically (see supplementary video 2).

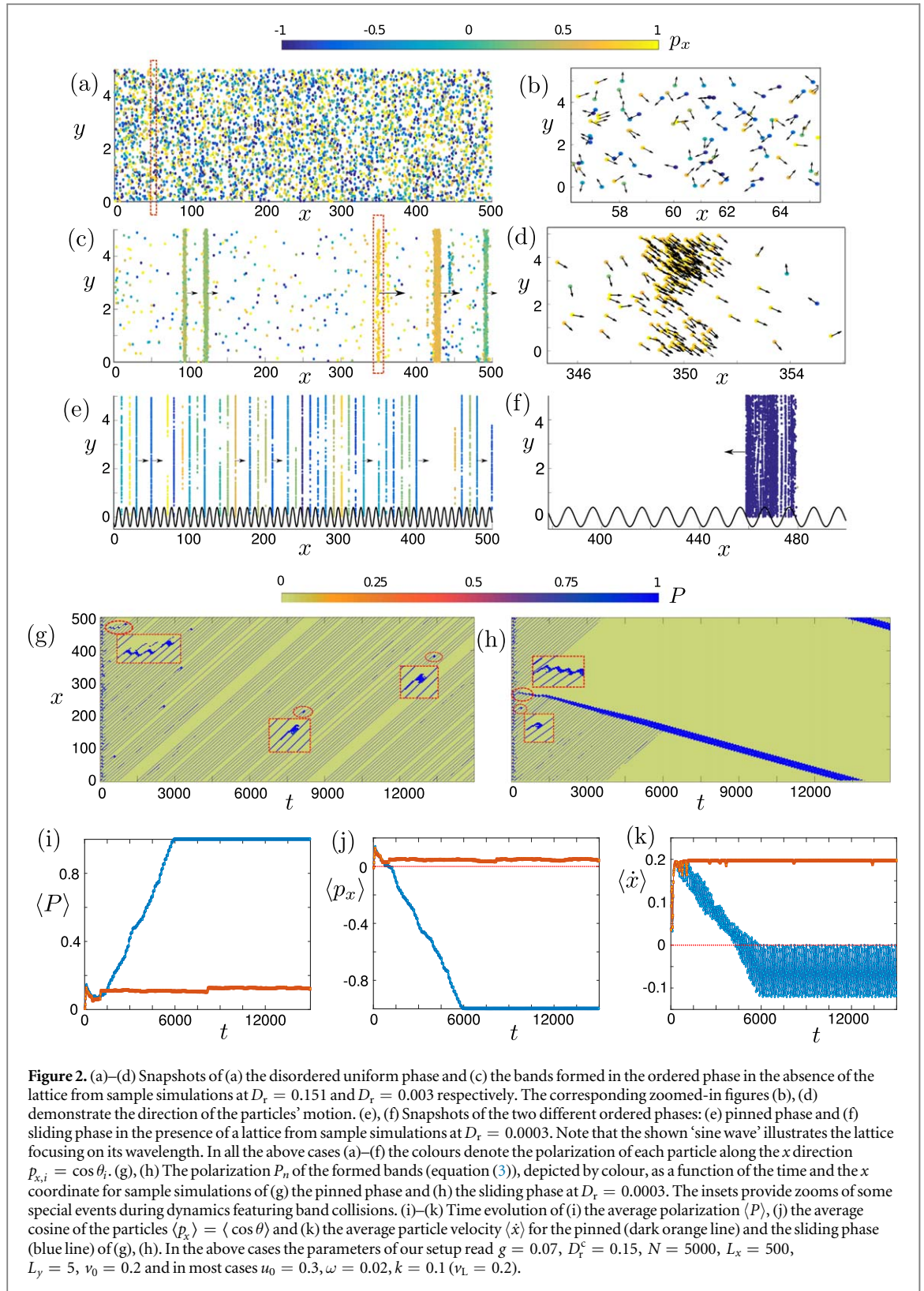
In the presence of the travelling wave (moving lattice) and at weak noise (we used $D_r = 3 \times 10^{-4} < D_r^c$) the behaviour of the bands may change dramatically. While very steep lattices of course pin the particles permanently to the lattice minima, leading to a state where all particles comove with the lattice, shallow lattices have little impact on the behaviour of the system and its tendency to form bands. In this latter regime, the lattice exerts a periodic force which essentially averages out before the particles move much. Thus, we here focus on moderate lattice depth ($u_0 = 0.3$) and lattice speeds comparable to that of the particles ($v_0 = v_L = 0.2$), so that the particles can occasionally overcome the potential maxima. In this regime, for sufficiently weak noise (here $D_r = 3 \times 10^{-4} < D_r^c$), particles form quickly bands, most of which are pinned to the lattice and thus co-move with it (figures 2(e), (g), (h), supplementary video 3). Note that the polarization of such bands

$$P_n = \sqrt{\left(\sum_{i \in \text{band } n} \cos \theta_i \right)^2 + \left(\sum_{i \in \text{band } n} \sin \theta_i \right)^2} \quad (3)$$

is almost unity even for small times and maintains this very high value during the time evolution (figures 2(g) and (h)). Occasionally, we observe that a band, assisted by the existing noise, changes direction and counterpropagates; it then soon collides with another band (see figures 2(g) and (h) insets for such collision events). Here, the two bands merge and form one larger band which in some cases becomes pinned and in other cases slides, still against the direction of motion of the lattice. In the latter case, the band soon encounters further

³ Consider the dynamics of free, not fully overdamped particles $m\ddot{\mathbf{r}}_i + \gamma\dot{\mathbf{r}}_i = \gamma v_0 \mathbf{p}_i$ yielding the solution $\dot{\mathbf{r}}_i(t) = v_0 \mathbf{p}_i (1 - e^{-\frac{\gamma}{m}t})$, i.e. the timescale on which a particle reaches the velocity $v_0 \mathbf{p}_i$ is $\approx \frac{m}{\gamma}$. We assume here that this time scale is much shorter than all other time scales in the system, including those applied by the travelling wave potential.

⁴ Choosing length and time units as $x_u = R$, $t_u = 1/D_r$ shows that the parameter space has five essential dimensions: the Peclet number $v_0/(D_r R)$ measuring the persistence length of the active particles (in bulk) in units of the particle radius, the reduced alignment rate $g/(\pi R^2 D_r)$, comparing the typical alignment rate of the particles with the Brownian decay of alignment, the reduced lattice depth $u_0/(\gamma R D_r)$ and the reduced frequency ω/D_r and wave vector kR of the lattice (and the density $\rho R^2 = 2$). The mass unit is implicitly fixed as $m_u = D_r/\gamma$ yielding an energy unit of $E_u = m_u x_u^2/t_u^2 = k T D_r R^2/D = 3kT/4$, where we have used the Einstein and the Stokes–Einstein–Debye relation.



bands and can in each case, either stop moving (get pinned to the lattice) or continue sliding. One might expect that this seemingly random result of the collision processes should ultimately lead back to a pinned state. Strikingly, however, in many simulations we observe cases where a band counterpropagates through the entire lattice and systematically consumes all other bands. The result is one macroband which contains most of the N particles and counterpropagates against the direction of lattice motion (figures 2(f), (h), supplementary video 4). Since the particles counterpropagate, even when viewed from the laboratory frame, with respect to the forces acting on a pinned particle in a minimum of the lattice, they feature an absolute *negative mobility*. Thus, we

observe a spontaneous reversal from a comoving state where most particles have followed the lattice to a counterpropagating state.

The striking difference between a finally pinned (figures 2(e) and (g)) and a finally sliding state (figures 2(f) and (h)) featuring a current reversal is further illustrated in figures 2(i)–(k). Here we observe that the mean polarization (averaged over all particles) $\langle P \rangle = \sqrt{\langle \cos \theta \rangle^2 + \langle \sin \theta \rangle^2}$ increases from the pinned state at short times to a value of almost one for the sliding macroband (figure 2(i)). It turns out (figure 2(j)) that $\langle p_x \rangle = \langle \cos \theta \rangle \approx -1$, meaning that the particles collectively self-propel against the direction of the lattice motion (still in the laboratory frame), i.e. along $-\mathbf{e}_x$. The average velocity of the particles is $\langle \dot{x} \rangle \approx v_L = 0.2$ (see also equation (1a)) for the pinned state (figure 2(k)) and acquires a negative value oscillating in time for the case of the sliding macroband.

4. Pinned and sliding solutions

We can get some first insight into the mechanism underlying the surprising counterpropagation of the bands by examining the single-particle dynamics in the zero noise limit. When projected to the x -axis equation (1a) reduces to

$$\dot{\tilde{x}} = \tilde{v}_x + \tilde{u}_0 \cos \tilde{x}, \quad (4)$$

where the Galilean transformation to the comoving frame $\tilde{x} = 2\pi(kx - \omega t)$ is used, with

$\tilde{v}_x = 2\pi k(v_0 p_x - v_L)$, $-1 \leq p_x = \cos \theta \leq 1$ and $\tilde{u}_0 = 2\pi k u_0$. This equation is well known as the overdamped limit of the equations of motion of e.g. the forced nonlinear pendulum [55], the driven Frenkel–Kontorova model [56] and the resistively shunted junction model of Josephson junctions [57]. It is known to attain two different kinds of solutions depending on the value of $\frac{\tilde{v}_x}{\tilde{u}_0}$. For $\left| \frac{\tilde{v}_x}{\tilde{u}_0} \right| \leq 1$ the system is in the so-called *pinned phase* where the particle cannot overcome the potential barrier \tilde{u}_0 and remains therefore trapped within one of its wells ($\langle \tilde{x} \rangle_t^\infty \rightarrow 0$), yielding an asymptotic time averaged velocity $\langle \dot{\tilde{x}} \rangle_t^\infty = v_L = \omega/k$. In the opposite case $\left| \frac{\tilde{v}_x}{\tilde{u}_0} \right| > 1$, the particle is fast enough to overcome the potential barrier separating the wells (or in the example of the pendulum to lead to a rotation) and thus the system exhibits a *sliding phase* where the particle permanently moves (slides or rotates) in one and the same direction with an oscillating velocity $\dot{\tilde{x}}$ [58] of period $T = \frac{2\pi}{\sqrt{\tilde{v}_x^2 - \tilde{u}_0^2}}$ [58] and an asymptotic time averaged velocity

$$\langle \dot{\tilde{x}} \rangle_t^\infty = \text{sgn}(v_0 p_x - v_L) \sqrt{(v_0 p_x - v_L)^2 - u_0^2} + v_L. \quad (5)$$

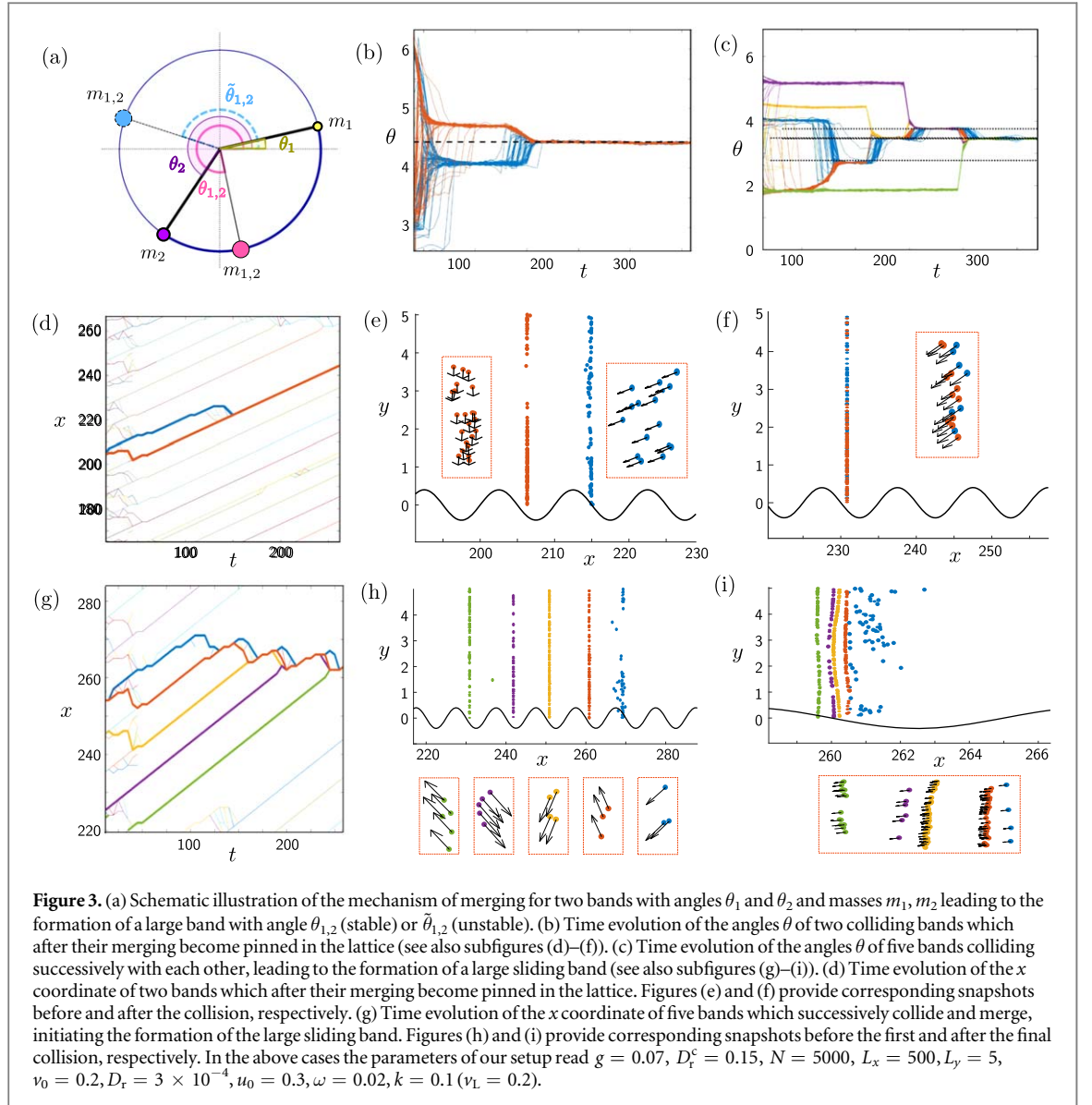
For the N -particle system (equations (1a) and (1b)) the particles' self-propulsion directions \mathbf{p}_i change due to alignment interactions (equation (1b)) and noise (equation (1a)). Hence, the projection of the particle speed onto the x -axis changes in time, so that the sliding condition $\left| \frac{\tilde{v}_x}{\tilde{u}_0} \right| > 1$ subsequently may and may not be fulfilled. In terms of \mathbf{p}_i , the sliding condition reads

$$-1 < p_{xi} < \frac{v_L - u_0}{v_0} \quad \text{or} \quad 1 > p_{xi} > \frac{v_L + u_0}{v_0}. \quad (6)$$

In the present parameter regime ($v_0 = v_L = 0.2$, $u_0 = 0.3$) sliding occurs for $p_{xi} < -\frac{1}{2}$, or $\theta_i \in \left[\frac{2\pi}{3}, \frac{4\pi}{3} \right]$. Thus roughly 1/3 of particles will initially be in the sliding phase. Importantly, all of these particles which can in principle slide, move *against* the direction of lattice motion (negative p_{xi}), i.e. sliding can only occur against the direction of lattice motion, as observed in figures 2(f), (h), (j) and (k). The main effect of rotational diffusion (noise in the particle orientations) consists in the smoothening of the pinned-to-sliding transition at $\left| \frac{\tilde{v}_x}{\tilde{u}_0} \right| = 1$.

5. Collisions of Vicsek bands

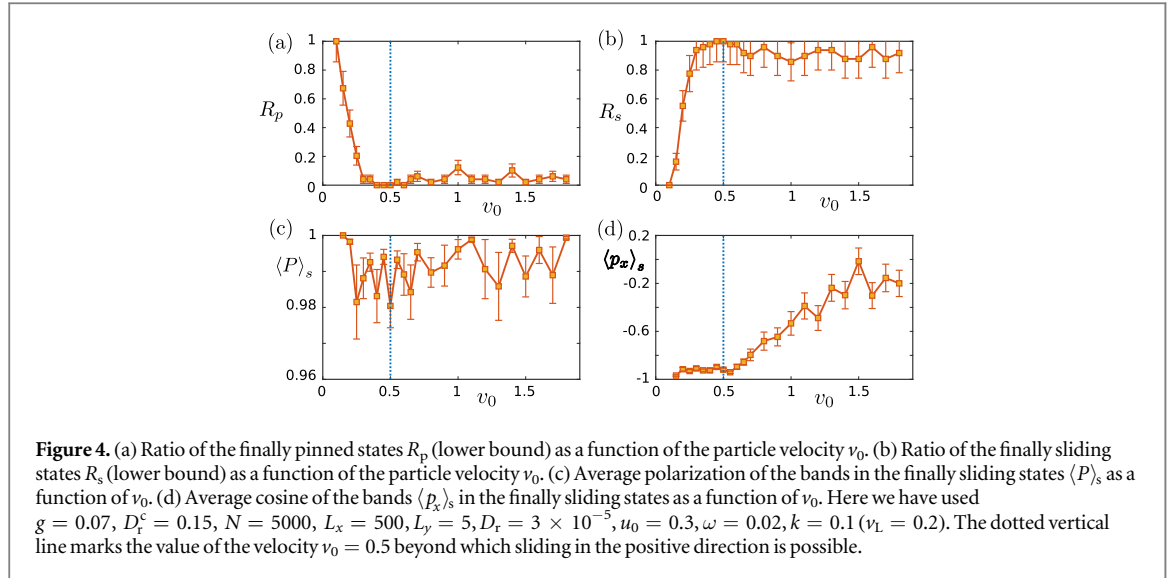
We now exploit these considerations regarding pinned and sliding states for single particles to understand the dynamics of the polarized bands in the lattice. In our simulations, shortly after their formation, the magnitude of the polarization of the individual bands quickly approaches a value close to one (figures 2(g) and (h)), i.e. most bands are moving with almost constant individual velocities relative to the lattice. Thus, in the absence of collisions, the bands essentially behave like single particles and are either pinned or slide through the lattice. To study band collisions, it is useful to assign effective 'masses' m_n to the bands representing the number of particles contained in the band. When two bands with polarization angles θ_1, θ_2 and masses m_1, m_2 collide, they usually merge into a larger band of total mass $m_{1,2} = m_1 + m_2$ (figure 3(a)) and average their polarizations. (Formally, there are two fixpoints of the orientational dynamics when two bands merge: one reads $\theta_{1,2} = \Theta_0$ with



$\Theta_0 = \frac{m_1\theta_1 + m_2\theta_2}{m_1 + m_2}$, the other one $\theta_{1,2} = \Theta_0 - 2\pi\frac{m_1}{m_1 + m_2}$; here the one lying within the smaller arc between θ_1 and θ_2 is stable and thus observed, the other one is unstable.)

In our simulations, the polarization direction of an isolated band can freely rotate (Goldstone mode); noise therefore creates a random dynamics of the band polarization direction. Once the polarization angle of an initially pinned band reaches a value $\theta_1 \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$ the band will move over a lattice barrier in the direction opposite to the lattice motion (figures 3(d), (e), (g) and (h)). Since the motion of the band towards an adjacent lattice site occurs on timescales which are short compared to the time noise needs to significantly change θ_1 , the band will typically feature an angle close to $2\pi/3$ or $4\pi/3$ when it encounters another pinned band. Depending on its relative orientation to the band it encounters, after the collision, its angle may either be out of the sliding interval (figure 3(b)), or, *similarly likely, may be deeper in the sliding interval* (figure 3(c)). In the latter case, the band continues counterpropagating through the lattice. Statistically, further collisions with other bands can be essentially viewed as a random walk of the band's polarization direction. Here, however, the effective mass of the band increases within each collision, corresponding to a decrease of the stepsize after each step. Hence, when the polarization of a band after a first few collisions is deeply in the sliding regime, i.e. $\theta \approx \pi$, the sliding of the band is highly robust against further collisions. This is why we have observed the emergence of a counterpropagating macroband consuming all other bands (figures 2(f) and (h)).

To understand the broad width of the counterpropagating macroband, it is instructive to resolve the collisions slightly further. When sliding bands collide, the positions of the contained particles do not fully mix up; rather, the resulting band features a substructure of microbands stacked one behind the other (figure 3(i)). This fact is responsible for the large width of the observed macroband (figure 2(f)) in the case of finally sliding states. This is because successive collisions typically result in a macroband with an average polarization close to



π , which is the centre of the sliding interval $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$. Thus after the collisions the involved particles move in the negative x direction with $\mathbf{p}_i \approx -\mathbf{e}_x$ (see figure 3(i)), a fact that prohibits them from mixing along the y direction. Conversely, band collisions leading to pinning do not induce a pronounced substructure. Here, the involved particles move significantly in the y -direction and therefore tend to mix in the course of the dynamics (figure 3(f)).

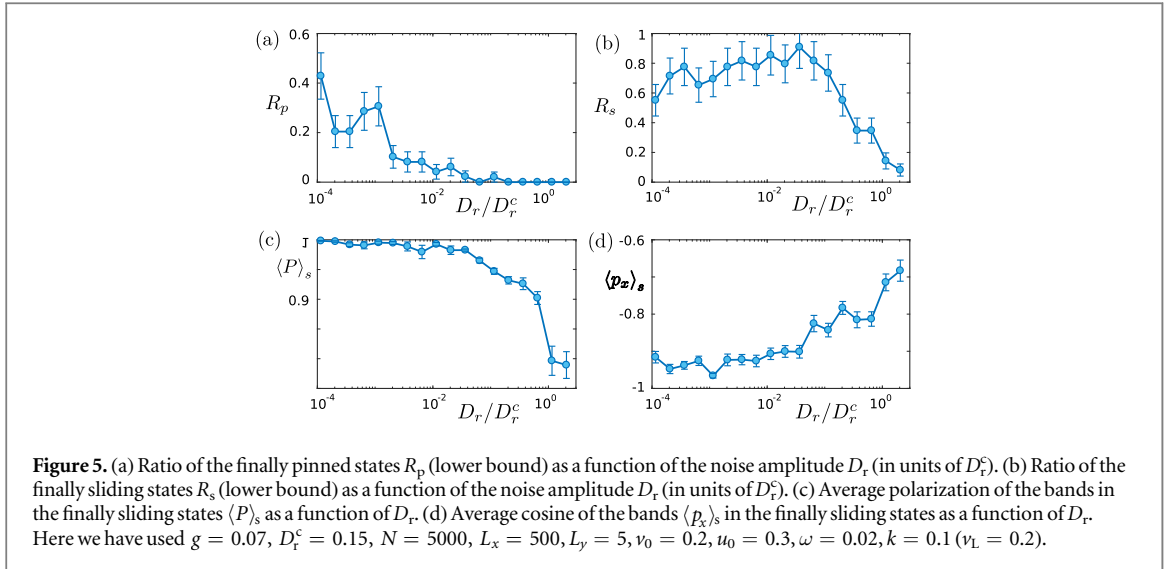
6. Effect of the particle speed

Having explored the mechanism leading to the dynamical reversal of the direction of motion of the particles in the moving lattice, we now ask how representative this scenario is. Here, we stay in the low noise regime ($D_r = 3 \times 10^{-5}$) and with our previous values for the lattice velocity $v_L = 0.2$ and height $u_0 = 0.3$ but vary the self-propulsion velocity of the particles. For $v_0 \leq 0.1$, we have $u_0 - v_L \geq v_0$ and hence sliding is not possible, whereas for $v_0 > 0.5$, where $v_0 > u_0 + v_L$, sliding can be achieved also in the positive direction (see equation (6)). In the complete interval $0.1 < v_0 \leq 0.5$ sliding is possible only against the lattice motion (negative direction) as discussed above. Within this interval, larger values of v_0 yield a larger interval of polarization angles leading to sliding (figures 4(a) and (b)). To specify this, we simulate 50 particle ensembles for each value of v_0 and count the number (ratio) of ensembles R_s which have reached a sliding and counterpropagating macroband and the corresponding ratio of ensembles R_p which have settled in an overall pinned configuration. Figure 4(b) shows that R_s increases monotonically in v_0 , crossing from a regime where most of the bands are pinned, even at late times ($v_0 \approx 0.1$) to a regime, where $R_s \approx 1$ ($v_0 \approx 0.5$). Thus, faster self-propulsion favours the emergence of a current reversal. (Note that the values of R_p , R_s shown in figures 4(a) and (b) should be viewed as lower bounds for the ratio of pinned and sliding states, as not all initial ensembles may have reached one or the other state at the end of our simulations.)

Complementary information about the finally sliding states, featuring a current reversal, is provided by figures 4(c) and (d). The final average polarization of these states $\langle P \rangle_s$ is for all v_0 very close to 1 (figure 4(c)), owing to the low noise which results in particles clustering to a macroband with a certain alignment. In contrast, the direction of this alignment, quantified by $\langle p_x \rangle_s = \langle \cos \theta \rangle_s$, is affected strongly by v_0 (figure 4(d)). For $0.1 < v_0 \leq 0.5$ we have $\langle p_x \rangle_s \approx -0.95$, indicating that within this interval the particles' velocities are all approximately aligned towards $-\mathbf{e}_x$ (as in figure 3(i)) and thus the particles counterpropagate at roughly their maximum velocity. This picture changes when $v_0 > 0.5$ where a sliding also in the forward direction becomes possible. Different realizations result in finally sliding states with a different alignment \mathbf{p}_i and thus their ensemble average $\langle p_x \rangle_s \rightarrow 0$ as v_0 increases, recovering the isotropy in the direction of sliding bands of the Vicsek model in the absence of a lattice.

7. Effect of the noise amplitude and the lattice wavelength

Rotational diffusion, whose strength is controlled by D_r , is crucial to initiate the emergence of sliding states; on the other hand, there is an upper critical noise strength D_r^c above which the Vicsek model does not show polar

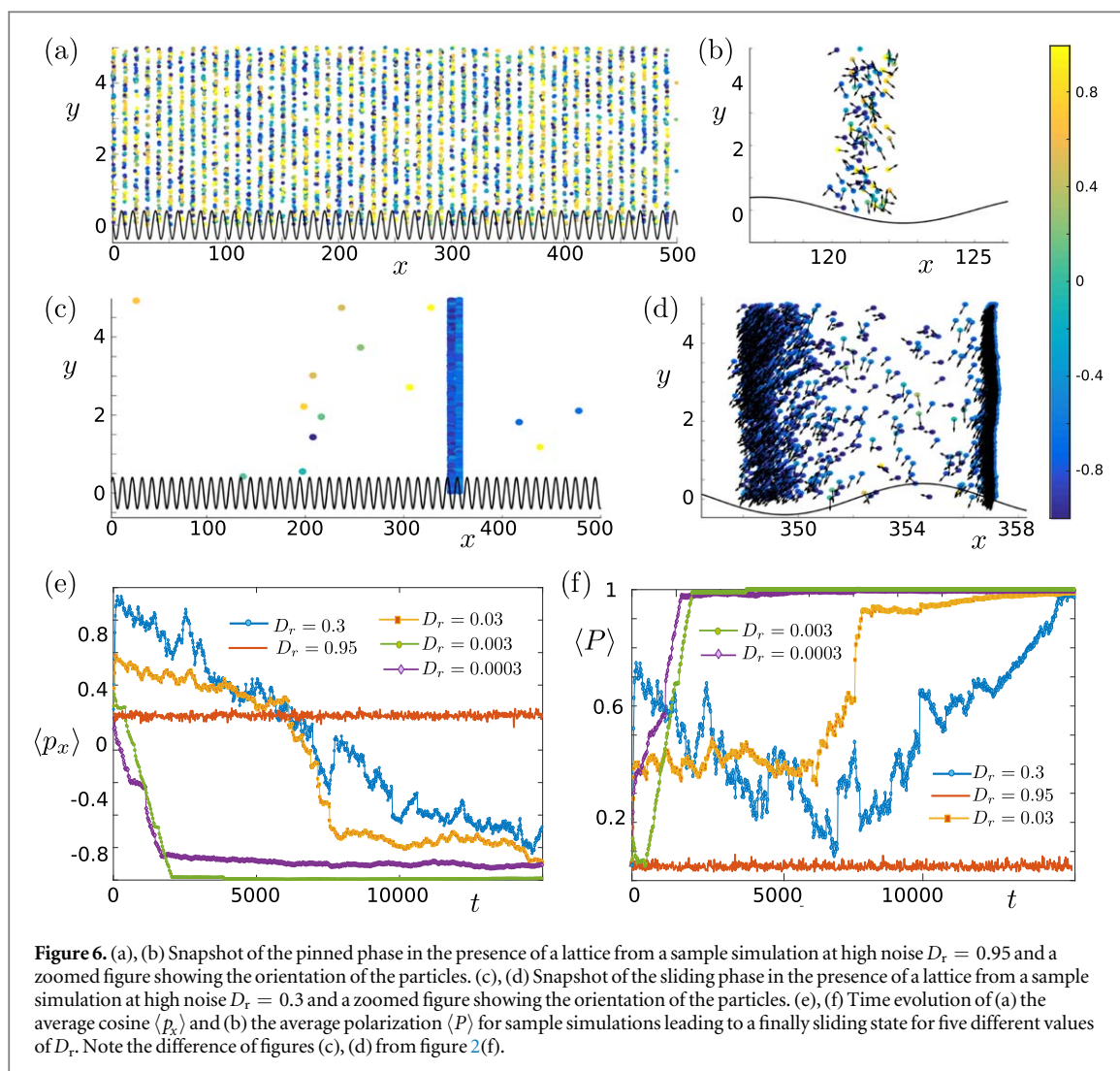


order, but is in the isotropic phase. We now systematically explore how the transition to the counterpropagating macroband is affected when changing D_r . For $u_0 = 0.3$, $v_0 = v_L = 0.2$ where sliding is possible only in the negative x direction, the ratio of states which are pinned at the end of our simulations R_p decreases as D_r increases (figure 5(a)) and finally approaches zero for $D_r \gtrsim 0.1D_r^c$. The reason for this behaviour is probably that larger noise turns the orientation of the polarization of initially pinned bands faster and therefore initiates sliding earlier (and more often). The respective ratio of finally sliding states R_s (figure 5(b)) increases only slightly as D_r increases from zero for $D_r \lesssim 0.05D_r^c$ and afterwards decreases tending towards zero. Physically, when D_r is too large, the polarization of a band may significantly change between each subsequent collision with other bands and may therefore leave the sliding regime before encountering another collision. Thus, for too strong noise, the emergence of a transition to a counterpropagating macroband is rather unlikely. The generic behaviour of the system for such high noise values is that of a mixture of individual particles, both in the pinned and in the sliding phase, whose polarization orientation changes fast in time, providing the picture of an overall disordered phase modulated by the existing potential wells (figures 6(a) and (b)). There is however still a possibility of obtaining a finally sliding state even in the high noise regime (figure 5(b)). Such states are significantly less polarized than the ones in the low noise regime (figures 5(c) and 6(c)) featuring also a larger variety in the direction of alignment, quantified by $\langle p_x \rangle_s$ (figures 5(d) and 6(d)). Furthermore, for such cases of high noise (e.g. $D_r = 0.3$, $D_r = 0.95$) the time evolution of both $\langle p_x \rangle_s$ and $\langle P \rangle_s$ is much slower (figures 6(e) and (f)) than the ones observed for lower noise values (e.g. $D_r = 0.0003$, $D_r = 0.003$), indicating the diffusive character of the dynamics expected for highly noisy systems.

Let us now briefly discuss the role played by the wavevector k of the travelling substrate, taken to be constant $k=0.1$ so far. We find that increasing k (decrease of lattice wavelength) clearly favours sliding, whereas its decrease increases the possibility of pinning, which may appear somewhat surprising as the angular-interval leading to sliding is independent of k as shown by equation (6). One likely reason for this is that for large k , where the lattice spacing is short, individual particles with appropriate orientation may cross a barrier before having enough time to align with the other particles in the well. Therefore angular-averaging remains incomplete and may lead to sliding of a small subband. In addition, once a sliding band has emerged, it might be more stable for large k . This is because for large k the force exerted by the lattice onto a band varies rapidly in space and essentially compactifies bands approaching a maximum of the lattice force. This enhances the density in the band, a fact that improves the corresponding alignment. On the contrary, for small k , noise can interrupt sliding relatively easily (particularly for cases of ‘stacked’ bands, discussed further above).

8. Conclusions

The present results provide a scheme allowing to control the typically highly irregular collective dynamics of polar active particles. In particular, we have seen that the bands occurring in the Vicsek model, which normally move in unpredictable directions and irregularly merge and split up can be tamed by applying a travelling wave-shaped potential, as can be realized e.g. using a micropatterned moving substrate or a traversing optical lattice. We find that while most particles in the system self-organize into polarized bands which comove with the lattice



at early times, they can later experience a remarkable reversal, initiated by the counterpropagation of a single band which subsequently consumes all other bands in the system. The asymptotic state is a strongly polarized macroband which predictably moves opposite to the direction of the motion of the external substrate. This behaviour is representative in a large parameter window and can be controlled e.g. by tuning the relative speed of the active particles and the lattice. Here, it would be interesting to consider particle ensembles with a characteristic velocity distribution rather than having identical velocities in the future. Our work may inspire further research of the interface between nonlinear dynamics and active matter and perhaps also applications regarding collective targeted cargo delivery using polar active matter.

Acknowledgments

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