On two Conjectures for Random Close Packings of Hard Spheres

Random close packings of hard spheres are notoriously difficult to treat mathematically. Practically all knowledge about them results from simulations of such systems. However, mathematical experience and intuition has led to empirical knowledge which can be cast in mathematical formulas. Two of them, for the case of identical spheres, can be found in Section 6.5.3 of the new book "Stochastic Geometry and its Applications" (3rd edition, 2013).

The first concerns the volume fraction V_V, i.e. the fraction of space occupied by the hard spheres, assuming that the sphere packing is infinite and spatially homogeneous. Numerical results already obtained by Scott (1960) say that $V_V = 0.6366$. And many simulations have produced ever and ever results close to 0.64. Since the random packing of hard spheres is an object so central and fundamental, it can be assumed that God's book contains a nice formula for V_V. So the author conjectured in 1998 that

$$V_V = 2/\pi.$$

The space between the hard spheres, called in the following 'pore space' is a random closed set of a rather complicated structure. A useful tool for its description is the spherical contact distribution function $H_s(r)$, the distribution function of the closest distance of a randomly chosen point of the pore space to the surface of its nearest sphere. In some way it describes the size of the (connected) pores.

Extensive simulations have led to the half normal distribution, i. e. to the following elegant formula for the probability density function $h_s(r)$ related to $H_s(r)$:

$$h_s(r) = \frac{2}{\sqrt{2\pi\sigma}} \exp(-\frac{r^2}{2\sigma^2})$$

with

$$\sigma = \frac{1 - V_V)2R}{3\sqrt{2\pi}V_V},$$

where R denotes the sphere radius R.

This formula turned out to be of value for the determination of volume fraction V_V and specific surface S_V of the so-called cherry-pit model. The spherical contact distribution function of this model can be easily expressed by the corresponding function of the hard sphere system forming the basis of the model, and V_V and S_V are closely related to $H_s(r)$.